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TE Module Design under Given Thermal Input: Theory and Design Example

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Abstract:	Established TE module theory provides direct calculation of the power output and conversion efficiency of a module if the temperature difference across the module is given. However, in many practical applications such as those using radioisotope or solar radiation as the heat sources, the temperature difference is usually unknown. Instead, only the thermal flux is given. In this paper, theoretical outline for TE module design under given thermal input is presented. It provides a convenient approach for module geometry optimisation based on given thermal input. The usefulness of the theory is demonstrated through a design study, in which an appropriate thermoelement length for solar thermoelectric application is determined through a trade-off between a requirement of longer length for establishing a larger temperature difference and a shorter length to produce a large power output.

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TE Module Design under Given Thermal Input: *Theory and Design Example*

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Abstract

Established TE module theory provides direct calculation of the power output and conversion efficiency of a module if the temperature difference across the module is given. However, in many practical applications such as those using radioisotope or solar radiation as the heat sources, the temperature difference is usually unknown. Instead, only the thermal flux is given. In this paper, theoretical outline for TE module design under given thermal input is presented. It provides a convenient approach for module geometry optimisation based on given thermal input. The usefulness of the theory is demonstrated through a design study, in which an appropriate thermoelement length for solar thermoelectric application is determined through a trade-off between a requirement of longer length for establishing a larger temperature difference and a shorter length to produce a large power output.

Introduction

Thermoelectric generator is a solid-state energy convertor that can convert heat into electricity. The theory for thermoelectric devices is well established [1-4]. It offers a useful tool for design and evaluation of a thermoelectric generator. In general, the power output and conversion efficiency of a thermoelectric generator can be determined using [1],

$$P = \frac{s}{(1+s)^2} \cdot \frac{\alpha^2 \cdot \Delta T^2}{R_i} \tag{1}$$

$$\phi = \frac{\Delta T}{T_H} \cdot \left(\frac{s}{(1+s) - \Delta T/2T_H + (1+s)^2/ZT_H} \right) \tag{2}$$

where $s\ (= R_L/R_i)$ is the ratio of the load resistance, R_L, to the internal resistance of thermoelectric generator, R_i. α is the Seebeck of coefficient; $\Delta T\ (= T_H - T_C)$ is the temperature difference across the thermoelectric generator and Z the figure-of-merit of thermoelectric materials. For given thermoelectric materials, equations 1 and 2 provide a direct calculation of the performances of a thermoelectric generator if the temperature difference across the generator is known. Generally, the temperature difference across a thermoelectric generator can be readily measured experimentally or assumed as a variable in the design process. As a result, they have been widely employed in the study of thermoelectric generators. An improved theoretical model is also available, which takes into consideration the effects of thermal and electrical contacts [5], providing a more realistic estimate of the generating performances in relation to the device's geometries. Similarly, the improved model also requires the temperature difference to be known or as an input viable. Although in many cases the temperature difference can be known in one way or another, there are other cases where only thermal energy input is given and the temperature difference across the generator needs to be determined based on materials properties, device geometries and operating conditions of the generator. The objective of this work is to provide a theoretical outline that can assist in the design of thermoelectric generators for a given thermal input.

Theoretical Outline

A simplified schematic of a thermoelectric generator is shown in Figure 1 (a), where a thermoelectric material is sandwiched between a heater at temperature, T_H, and a heat sink at temperature, T_C. The two ends of the thermoelectric material are electrically connected to an external load, R_L through a switch. Its equivalent electric circuit is shown in Figure 1(b). For a given thermal input, $\dot{Q}$, and considering an ideal case where the heat loss is neglected, the temperature difference across the generator when operating in an open circuit (i.e., the generator is not connected to the load) can be determined using Fourier's law,

$$\Delta T_o = \frac{\dot{Q}}{\lambda \cdot (A/l)} \tag{3}$$

where, λ is the thermal conductivity of the thermoelectric material and A/l is the ratio of the cross-sectional area, A, to the length, l, of the thermoelectric generator. Usually, a thermoelectric generator consists of multiple thermoelements. The cross-sectional area A in this case is the total area of all thermoelements. It is to be noted that the temperature difference determined using equation 3 is only valid for the open circuit condition. When the generator is connected to a load, a close circuit is formed and the temperature difference across the generator will be reduced depending on the ratio of the load resistance to the internal resistance of the generator, R_L / R_i. Using a relationship reported in [6],

$$\frac{\Delta T_o}{\Delta T} = 1 + ZT_M \tag{4}$$

where,

$$T_M = \frac{(1+2s)T_H + T_C}{2(1+s)^2} \tag{5}$$

Consequently, the temperature difference across the thermoelectric generator under the closed circuit condition can be expressed as,

$$\Delta T = \frac{\dot{Q}}{(1+ZT_M)\cdot\lambda\cdot(A/l)} \tag{6}$$

Taking into account the effect of thermal and electrical contacts, the power output of a thermoelectric generator can be expressed as [5],

$$P = \frac{s}{(1+s)^2}\frac{\alpha^2}{\rho}\frac{A\cdot\Delta T^2}{(n+l)(1+2rl_c/l)^2} \tag{7}$$

where, l_c is the thickness of ceramic plates (i.e., thermally conducting but electrically insulating layers in the thermoelectric generator. n and r are referred to as electrical and thermal contact parameters, respectively [7]. Replacing ΔT in equation 7 using equation 6, the power output can be expressed as,

$$P = \frac{s}{(1+s)^2} \cdot \frac{Z}{(1+ZT_M)^2} \cdot \frac{\dot{Q}^2}{\lambda} \cdot \frac{l}{A} \cdot \frac{l}{(n+l)(1+2rl_c/l)^2} \quad (8)$$

The conversion efficiency of the generator can be expressed as,

$$\eta = \frac{P}{\dot{Q}} = \frac{s}{(1+s)^2} \cdot \frac{Z}{(1+ZT_M)^2} \cdot \frac{\dot{Q}}{\lambda} \cdot \frac{l}{A} \cdot \frac{l}{(n+l)(1+2rl_c/l)^2} \quad (9)$$

It can be seen that calculation of the power output and conversion efficiency using equations 8 and 9 does not require the knowledge of the temperature difference. Instead, it is determined from the given thermal input.

Design Example

A schematic solar thermoelectric system is shown in Figure 2. Assume that the solar absorber has an area of 1 m^2 and the thermal power produced under AM1.0 (i.e., 1000 W/m^2) by the absorber that has a efficiency of 60% is 600 W. It is also assumed that the thermal power does not change with the surface temperature of the absorber (strictly speaking, this simplification is not usually valid. Nevertheless, it is employed to focus on the thermoelectric aspect). It can be seen from equation 6 that the temperature difference across the generator will be large if the thermoelectric generator has a small A/l. On the other hand, for a given ΔT, a large A/l is required to obtain a large power output. Clearly, a key task in the design of a thermoelectric generator is to select an appropriate A/l. In Figure 3 is shown the electrical power output generated by thermoelectric generator as a function of l for different A/l. The results were calculated based on Bi_2Te_3 thermoelectric modules, which has a typical Seebeck coefficient of $200\times10^{-6}\ V/K$, electrical conductivity of $10^{-5}\ \Omega\cdot m$, thermal conductivity of $1.5\ W/m\cdot K$; the thickness of ceramic layer is 0.7 mm; the electrical and thermal contact parameters are n = 0.1 mm and r = 0.2 [7], respectively; The thermoelectric generator operates in the maximum power mode (i.e., s=1) and there is no heat losses from both the solar absorber and thermoelectric generator. It is interesting to note that the plots of power output v.s. thermoelement length resemble typical I–V characteristics of the transistors. For a given A/l, the power output increases significantly with an increase in l initially but reaching saturation over a longer length. For a given l, the power output increases with a decrease in A/l. For example, selecting $A/l = 2656$mm will results in a temperature difference of 100 K across the thermoelectric generator. A power output of ~24 W can be obtained for $l = 7$mm and a corresponding total cross-sectional area is $A \approx 1.9\times10^4\ \text{mm}^2$. This is equivalent to a total area of 36 commercial Bi_2Te_3 modules, each of which has 127 thermocouples with a thermoelement cross-sectional area of 1.4 mm x 1.4 mm. If the number of modules is reduced to 18 (i.e. A/l is reduced to 1244 mm), the temperature difference across the thermoelectric generator will be 200 K. This will result in an increase of power output to ~ 46 W. It is to be noted that this power level may not be achievable in practice due to significant heat losses from the solar absorber at high temperatures. As a result, the power output is a trade-off between a small A/l for obtaining large power output and a large A/l for minimizing heat losses. Nevertheless, this ideal case demonstrates that increasing the number of thermoelectric modules does not necessarily lead to an increase in power output. This is particularly true for the cases when the temperature difference across thermoelectric generator is determined by the given thermal input.

Conclusion

The theory presented in this paper enables calculation of the power output and conversion efficiency of a thermoelectric generator from a given thermal input. This provides a complementary formulation to the conventional theory, which requires the knowledge of the temperature difference across a thermoelectric generator. The usefulness of this new formulation is demonstrated by a case study of geometrical effect on the power output of the thermoelectric generator in a solar thermoelectric system. Although the example given is based on an ideal case where all heat losses in the system were neglected, the results highlight role of thermoelement geometry in the design of optimized thermoelectric system.

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Caption of Figures

Figure 1. Schematics of a simplified thermoelectric generator. a) thermoelectric generator with an internal resistance R_i connected to a external load R_L; b) equivalent circuit for thermoelectric generator connecting to a load.

Figure 2. Schematic solar thermoelectric system. The solar radiation is converted into heat by the absorber and transferred through thermoelectric generators into circulating water for heat and power co-generation.

Figure 3. The power output of thermoelectric generator as a function of thermoelement length, l, for different ratios of the total cross-sectional area to the thermoelelment length, A/l.

Figure 1a
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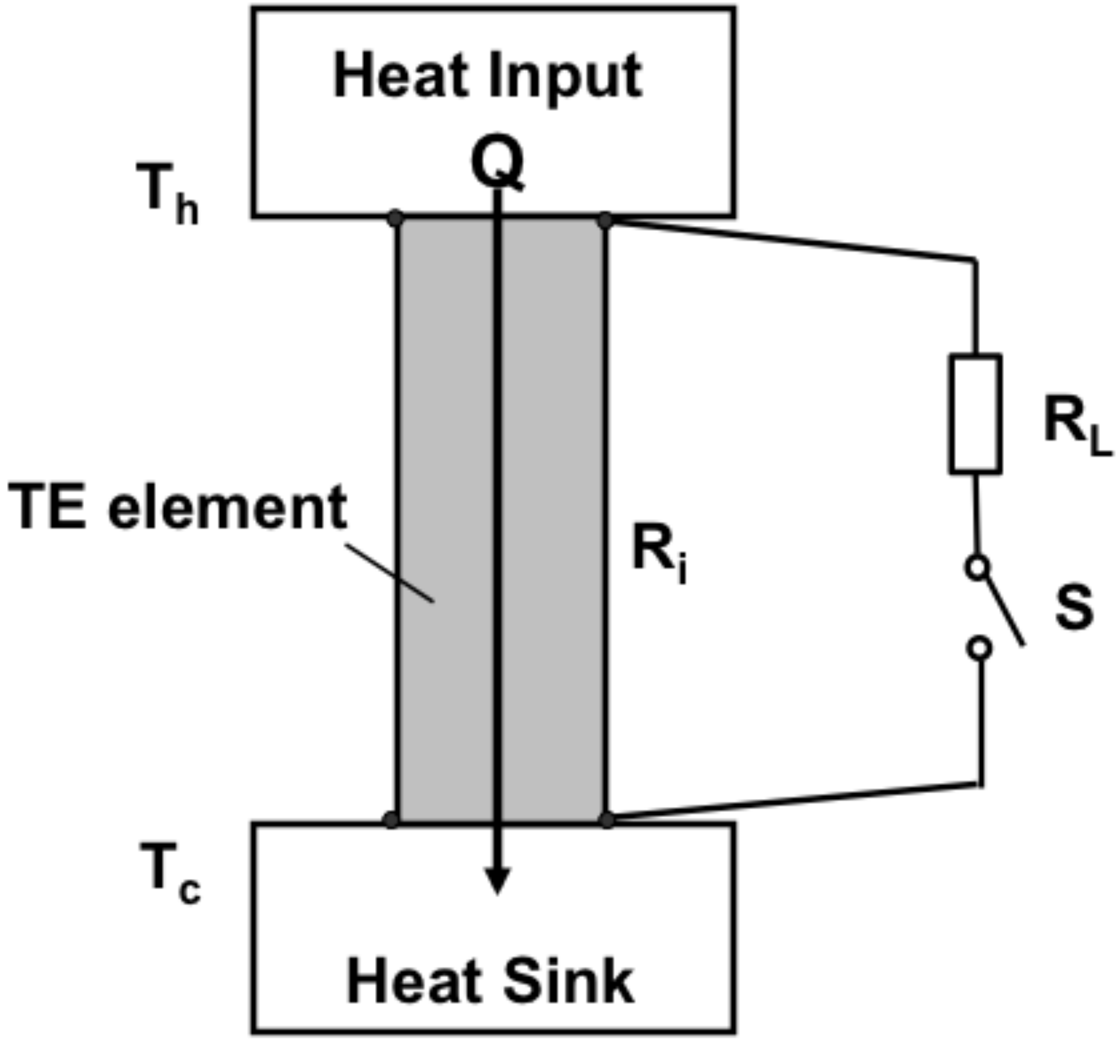

Figure 1b
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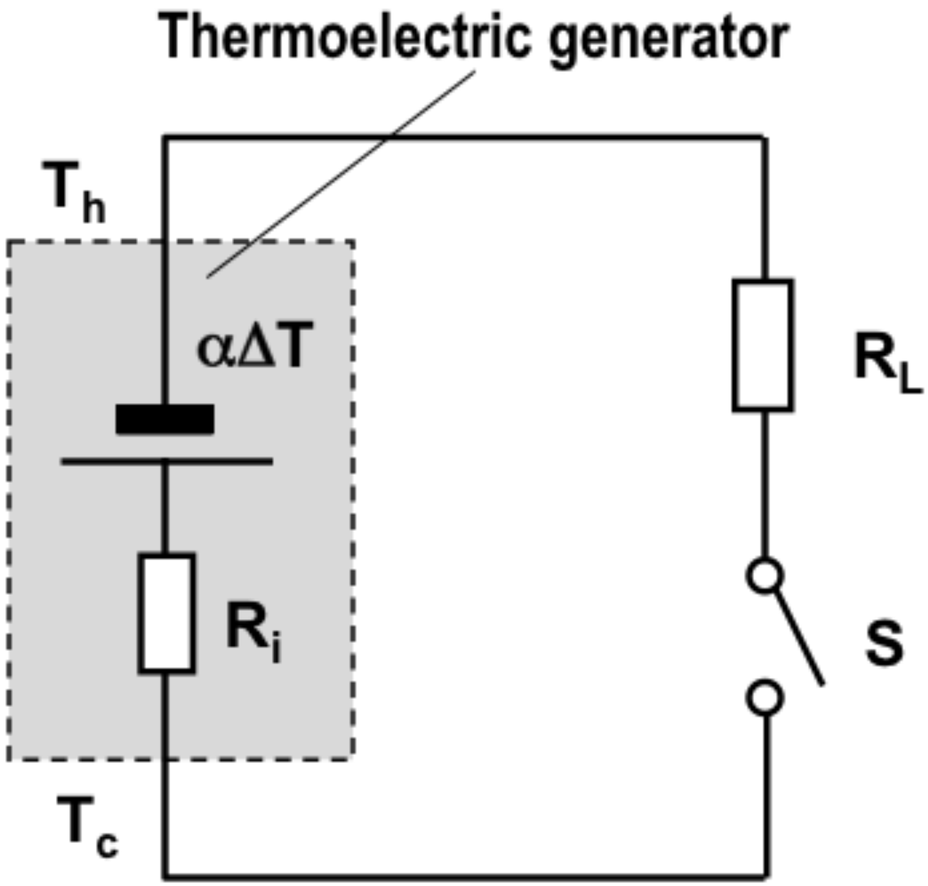

Figure 2
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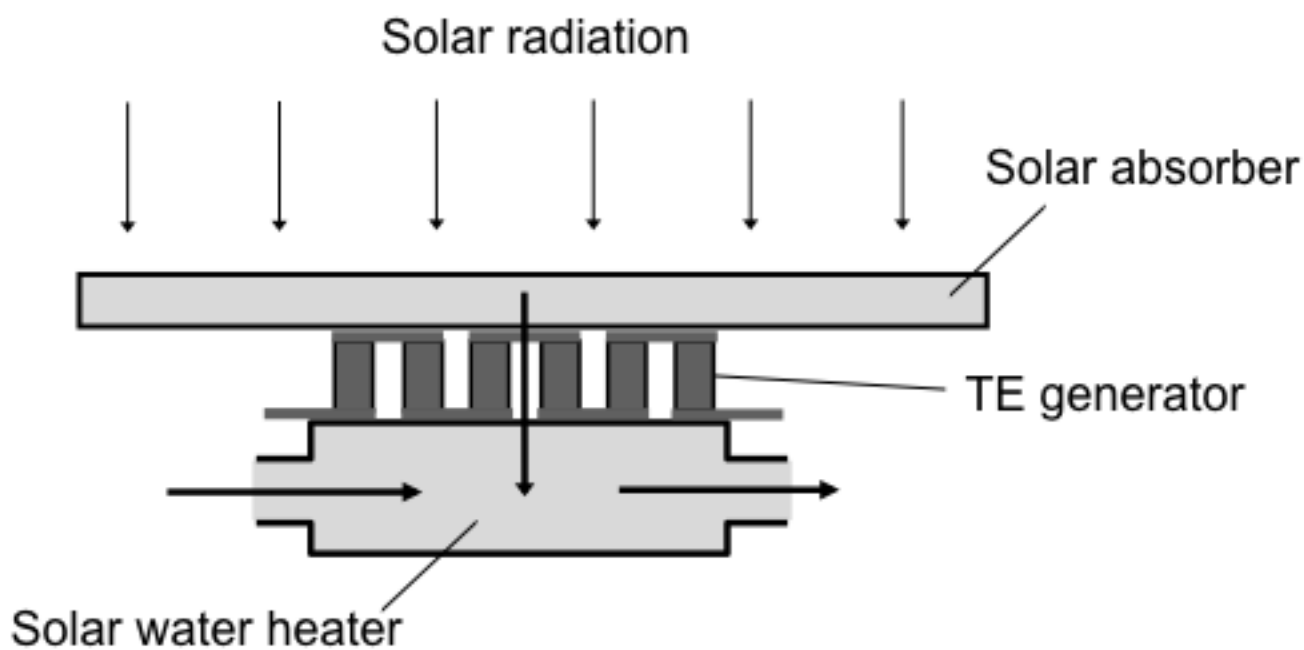

Figure 3
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